Overconstrained Wheeled Vehicles: A Simpler Rocky 7–The Kinematic Car

***Neeraj Singh Gautam and **Prashant Awadhiya**
Department of Mechanical Engineering,
Government Engineering College , Raipur (C.G.) 492 010 India.
E-mail: *iasgautam@rediffmail.com

Abstract

This paper examines an application of the controllability theory to the case of MMDA systems where the individual models are essentially nonlinear. First, the concepts are more easily illustrated in this nonlinear setting in the context of an example of the Mars Rover. The Mars Rover is itself an important engineering application, and this paper contributes to understanding how to design an appropriate control structure. This paper examined in detail the controllability of a simple model of the Rocky 7 Mars Sojourner. Some initial steps towards a motion planning have been outlined and preliminary results for control of such vehicles are given. The extension of Chow's theorem is used to show conditions under which vehicles like the Mars rover are controllable. Moreover, variations of the rover are used to illustrate that controllability of the individual models, which make up a multiple model driftless affine system, is not sufficient to guarantee controllability of the overall multiple model driftless affine system. Section 3.1 discusses the full input space for a six-wheeled, fully actuated system. The potential importance of the vehicles discussed in Section 3 in future planetary exploration missions indicates the need for more in-depth analysis of stabilization. Future work will investigate algorithms for stabilizing the multiple model systems of Definition 3.2.

Index Terms— Mobile robots, nonholonomic motion planning, overconstrained vehicles*,* obstacle avoidance, path tracking, vehicle control, *MMDA systems*

1. Introduction: The Rocky 7 Mars Sojourner

Most mobile robots use wheels since they provide one of the simplest means for mobility. Wheels impose nonholonomic constraints on a vehicle's motion, and thus the subject of control and motion planning for nonholonomic wheeled vehicles has been widely pursued [1,2, 3]. In unstructured terrain where there are many obstacles, legged robots may be used [4], although then one is faced with challenges that are associated with balance. Therefore, the simplicity of a wheeled robot makes it an appealing alternative to legged robots even in somewhat rough terrains.

In order to operate in moderately rough terrains without resorting to the inherent complexity of legged system design, "overconstrained" wheeled vehicle designs have been proposed. The most famous example of such a vehicle is the Sojourner robot deployed during the Mars Pathfinder mission. Figure 1 shows the "Rocky 7," a prototype for future Mars rover vehicles whose suspension and wheel kinematics are basically identical to the Sojourner vehicle. The Rocky 7 employs six wheels, with both front wheels independently steered and all six wheels independently driven, making it an eight-dimensional control space.

Figure1: Photo of Rocky 7 mars rover prototype

The rear wheels on each side are coupled through a "bogey" linkage mechanism that helps the vehicle negotiate obstacles that are up to 1.5 times the wheels' diameter. It has been shown that standard nonholonomic motion planning and control theories cannot be applied to this vehicle. To understand the key issues that are addressed in this paper, consider a highly simplified model of the Rocky 7 vehicle(Figure 2(b)).

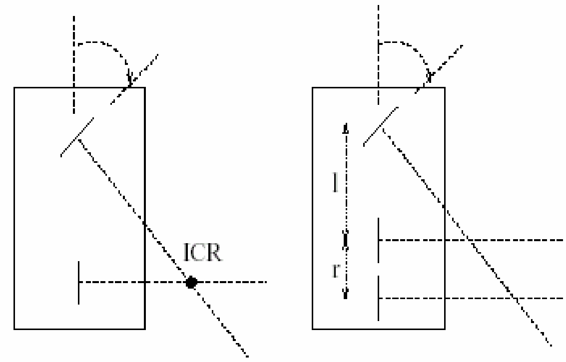

Figure 2: (a) kinematic car; (b) simplified Rocky 7.

In this simplified model vehicle, the Simplified Rocky 7, hereafter referred to as SR7, operates on flat terrain. To realize the model of Figure 2, each pair of Rocky 7 wheels is conceptually "collapsed" into a single wheel, in a manner similar to that of conventional models of the classical kinematic car (Figure 2(a)). Furthermore, assume that only the front wheel is actuated. While highly simplified, this model captures many of the essential features and challenges of overconstrained wheeled vehicles. Obviously, the fact that Rocky 7 operates in non-planar terrain and has additional wheel actuation will pose further complexities. The motion any every planar body can be characterized at each instant by its Instantaneous Center of Rotation (ICR). In the classical kinematic car model (Figure 2(a)), the assumption that the wheels do not slip defines an instantaneous center of rotation at the intersection of the lines that are collinear with the two wheel axes. Note that the presence of an additional wheel leads to an overconstraint and kinematic constraints alone can not be used to determine the ICR of the SR7 vehicle in Figure 2(b).

The dynamics of such a vehicle are overconstrained because both back wheel axes prevent the automobile wheels from sliding sideways, which means there are two parallel constraints In order to satisfy these constraints, the vehicle must always drive straight forward, and if the front wheel is turned, the vehicle must stay still. Therefore, even in the kinematic case, finding the equations of motion governing the system is a nontrivial task.

2. A Simpler Rocky 7–The Kinematic Car

The kinematic car as shown in Figure 3 has been studied extensively as an example of a system with nonholonomic constraints. It is used here as an example of how to apply Chow's theorem to a smooth nonholonomic system. It has been shown by Murray et al. [70] that sinusoidal inputs at integral frequencies will produce Lie bracket motions associated with parallel parking and that global stabilization can be obtained using feedback. Assume that the car is driven by the front wheels and that the wheels roll without slipping.

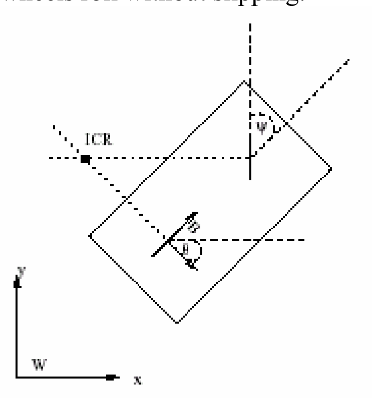

Figure 3: The kinematic car

The constraints associated with zero wheel slip are

$$\begin{bmatrix} \sin(\theta+\psi) & -\cos(\theta+\psi) & -l\cos(\psi) & 0 & 0 \\ \sin(\theta) & -\cos(\theta) & 0 & 0 & 0 \\ \cos(\theta+\psi) & -\sin(\theta+\psi) & -l\sin(\psi) & R & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = 0.$$

where R is the radius of the front wheels, x and y are the coordinates of the body frame B, θ is the angle between the B frame and the W frame, ϕ is the front wheel angle, and φ is the angle between the front wheels and the B frame. First find g_1 and g_2 that annihilate these constraints, and then take their Lie brackets to find the distribution

$$g_1 = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{1}{l}\tan(\psi) \\ 0 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$g_3 = [g_1, g_2] = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{l\cos^2(\psi)} \\ 0 \end{bmatrix} \qquad g_4 = [g_1, g_3] = \begin{bmatrix} -\frac{\sin(\theta)}{l\cos^2(\psi)} \\ \frac{\cos(\theta)}{l\cos^2(\psi)} \\ 0 \\ 0 \end{bmatrix}$$

so $\overline{\Delta}$ = span{g_1, g_2, g_3, g_4} = R^4, which implies that the system is locally controllable.

3.1 Six Wheels and Two Steerable Wheels

First consider a full kinematic model of the Rocky 7 Sojourner robot. It has two steerable front wheels (inputs u_1, u_2) and all six of its wheels are driven (inputs $u_3, \ldots, u_8$). Assume that the vehicle is on flat ground with only variations in the coefficient of friction altering the contact state. Idealize the steering of the wheels as a rotation about a vertical axis. In this model, there are a total of 12 nonholonomic constraints on the system, with each wheel contributing a "no side-ways slip" constraint and a "no rolling" constraint. Clearly, not all of these constraints can

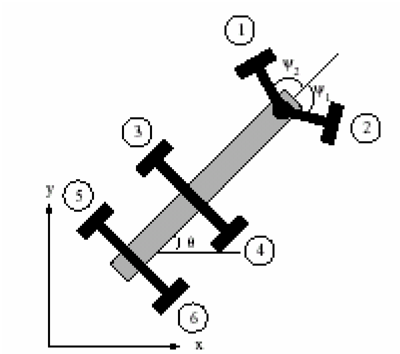

Figure 4: 6 driven wheels, 2 steerable wheels

be simultaneously satisfied except in nongeneric cases: 1) the two rear axles are parallel, and therefore can only accommodate forward motion without slipping, and 2) there is no a prior reason to believe that the inputs u_i will produce mutually compatible velocities. Thus, the system is overconstrained. Applying the PDM to this system, one gets

$$\binom{12}{3} = \frac{12!}{9!3!} = 220$$

kinematic states. That is, there are 220 different combinations of slipping motions. This number is daunting both from a complexity and from practical standpoint. To make progress tractable on the analysis front, reduce the number of inputs by introducing "matching" constraints. Observe that for any choice of u_1 and u_3 one can choose the other u_i inputs to be kinematically compatible with the motion produced by u_1 and u_3. Therefore, reduce the dimension of the input space by requiring the following to hold:

$$\begin{aligned} u_6 &= Ad^{[2]}_{g_{63}} u_3 \\ u_7 &= Ad^{[2]}_{g_{73}} u_3 \\ u_8 &= Ad^{[2]}_{g_{83}} u_3 \\ u_5 &= Ad^{[2]}_{g_{53}} u_3 \\ u_4 &= Ad^{[2]}_{g_{43}} u_3 \\ u_2 &= Ad^{[2]}_{g_{12}} u_1 \end{aligned} \qquad (1)$$

where $Ad^{[k]}_{g_{ij}}$ is the k^{th} component of the Adjoint operator of the rigid body trans-formation going from frame i (associated with the point where the input ui acts on the system) to frame j.

3.2 Reduction

In this example, the front wheel is driven u1 and steered by u2, while u3 and u4 drive the middle and back wheels respectively. Setting u3 = u1 and u4 =

u1 produces the desired reduction in Equation (1). Additionally, the front wheel is always assumed to be in contact with the ground1.

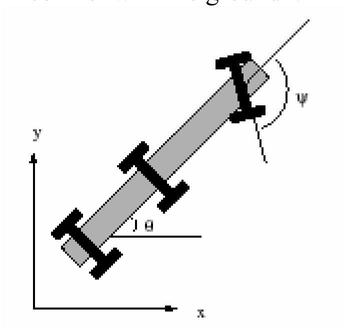

Figure 5: 1 driven and steered wheel, 2 passive wheels

Assume that this is driven by the front wheels and that the wheels roll without 1143 slipping. The constraints associated with zero wheel slip are

$$\Omega(q)\dot{q} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \sin(\theta+\psi) & -\cos(\theta+\psi) & -l\cos(\psi) & 0 & 0 \\ \cos(\theta+\psi) & -\sin(\theta+\psi) & -l\sin(\psi) & R & 0 \\ \sin(\theta) & -\cos(\theta) & 0 & 0 & 0 \\ \sin(\theta) & -\cos(\theta) & r & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = 0.$$

where R is the radius of the front wheels, x and y are the coordinates of the body frame B, , θ is the angle between the B frame and the W frame, ϕ is the front wheel angle, and φ is the angle between the front wheels and the B frame. The constraints (ϖ_1, ϖ_2, ϖ_3, ϖ_4) are the front wheel rolling, front wheel side ways slipping, middle wheel side ways slipping, and back wheel side ways slipping. It is easy to check that for φ ≠ 0 this only has a solution of $\dot{q} = 0$. Moreover, if ϕ ≠ 0 it has no solution when φ ≠ 0.

If the vehicle configuration is $q = [x, y, \theta, \psi]^T$ and the controls u_1 and u_2 are associated with the drive and steering velocities respectively, the vehicle's governing equations of motion are

$$\dot{q} = g_{\sigma_1}(q)u_1 + g_3(q)u_2 \qquad \sigma_1 : (q, t) \rightarrow \{a, b\}$$

$$g_a = \begin{bmatrix} \cos(\psi)\cos(\theta) \\ \cos(\psi)\sin(\theta) \\ \frac{1}{l}\sin(\psi) \\ 0 \end{bmatrix} \quad g_b = \begin{bmatrix} \cos(\theta)\cos(\psi) - \frac{r\sin(\theta)\sin(\psi)}{l+r} \\ \cos(\psi)\sin(\theta) + \frac{r\cos(\theta)\sin(\psi)}{l+r} \\ \frac{1}{l+r}\sin(\psi) \\ 0 \end{bmatrix} \quad g_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The function which determines the switching boundaries is:

$$\Psi(q) = \left(\frac{F_1\mu_1}{F_2\mu_2}\right)^2 \left(\frac{l-r}{r}\right)^2 - 1$$

where F_i are the normal forces above the middle axis and back axis, and the μ_i are the coefficients of friction at the two rear wheel contacts. Controllability is determined by the rank of the distribution:

$$\Delta = \{g_3, g_{\sigma_1}, [g_3, g_{\sigma_1}], [[g_3, g_{\sigma_1}], g_{\sigma_1}\}) \qquad (2)$$

Computing accordingly, we get $[g\sigma_1, g_3]$ =

$$co\left\{ \begin{bmatrix} \cos(\theta)\sin(\psi) \\ \sin(\theta)\sin(\psi) \\ \frac{-\cos(\psi)}{l} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{r\cos(\psi)\sin(\theta)}{l+r} + \cos(\theta)\sin(\psi) \\ \frac{-r\cos(\theta)\cos(\psi)}{l+r} + \sin(\theta)\sin(\psi) \\ -\left(\frac{\cos(\psi)}{l+r}\right) \\ 0 \end{bmatrix} \right\}$$

$$[g_{\sigma_1}, [g_{\sigma_1}, g_3]] = co\left\{ \begin{bmatrix} \frac{-1}{l}\sin(\theta) \\ \frac{1}{l}\cos(\theta) \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{-1}{l+r}\sin(\theta) \\ \frac{1}{l+r}\cos(\theta) \\ 0 \\ 0 \end{bmatrix} \right\}$$

Now, using the algebraic equivalent for co{·, ·}, we can evaluate the determinant of Equation (2):

$$\begin{aligned} &\det [g_3, co\{g_{1a}, g_{1b}\}, [g_3, co\{g_{1a}, g_{1b}\}], [[g_3, co\{g_{1a}, g_{1b}\}], co\{g_{1a}, g_{1b}\}]] \\ &= \det [g_3, co\{g_{1a}, g_{1b}\}, co\{[g_3, g_{1a}], [g_3, g_{1b}]\}, \\ &\qquad co\{[g_{1a}, [g_{1a}, g_3]], [g_{1b}, [g_{1b}, g_3]]\}] \\ &= \det [g_3, \delta_1 g_{1a} + (1-\delta_1)g_{1b}, \delta_2[g_3, g_{1a}] + (1-\delta_2)[g_3, g_{1b}], \\ &\qquad \delta_3[g_{1a}, [g_{1a}, g_3]] + (1-\delta_3)[g_{1b}, [g_{1b}, g_3]]] \end{aligned}$$

where δi ε [0, 1] for i = 1, 2, 3. The determinant is

$$\frac{(l + r\delta_3)(2l + r(\delta_1 + \delta_2) + r(\delta_2 - \delta_1)\cos(2\psi))}{2l^2(l+r)^2}$$

which equals 0 only if δ3 < 0 and these values are not admissible. Hence, the vehicle is always STLC, as expected. Physically, this result implies that the vehicle remains locally controllable even as the status of slipping wheel alters unexpectedly.

4. Conclusions

In this paper an example of the Mars Rover has been presented which is relevant to the analysis of driftless overconstrained mechanical systems. The power dissipation methodology for finding tractable governing equations for overconstrained mechanical systems has been revived. Understanding the issue of controllability is often a first step toward understanding how to control a class of nonlinear systems. It has been revealed that standard nonholonomic motion planning and control theories cannot be directly applied to the important class of overconstrained wheeled vehicles. Some initial steps towards a motion planning have been outlined and preliminary results for control of such vehicles are given. The extension of Chow's theorem is used to show conditions under which vehicles like the Mars rover are controllable. Moreover, variations of the rover are used to illustrate that controllability of the individual models, which make up a multiple model driftless affine system, is not sufficient to guarantee controllability of the overall multiple model driftless affine system.

5. Acknowledgement

Words are boundless to express my heartfelt gratitude and thanks to Dr. S. Sanyal and Mr. S. P. S. Matharu for their invaluable guidance, encouragement, inspiriting suggestions and whole hearted co-operation to us not only in completing this work to almost perfection but also in due time. I am also grateful to the Faculty of Deptt. Of Mechanical Engineering, Engineering College, Raipur to help me time to time.

6. Bibliography:

[1] A.M. Bloch, P. S. Krishnaprasad, J.E. Marsden, and R.M. Murray. Nonholonomic mechanical systems with symmetry. Arch. Rational Mech. Anal., 136 (1):21–99, 1996.

[2] R.M. Murray and S.S. Sastry. Steering nonholonomic systems in chained form. In Proc. Int. Conf. on Decision and Control (CDC), pages 1121–1126, 1991.

[3] R.M. Murray and S.S. Sastry. Nonholonomic motion planning: Steering using sinusoids. IEEE Transactions on Automatic Control, 38:700–716, 1993.

[4] S.M. Song. and K.J. Waldron. Machines that walk: the Adaptive Suspension Vehicle. MIT Press, 1989.

[5] K.J. Waldron and C.J. Hubert. Control of contact forces in wheeled and legged o_-road vehicles. In Preprints, 6th Symp. on Experimental Robotics, pages 265–274, Syndney, Australia, March 1999.

[6] T. D. Murphey and J. W. Burdick. Nonsmooth controllability and an example. In Proc. IEEE Conf. on Decision and Control (CDC), Washington D.C., 2002.

[7] R.M. Murray, Z. Li, and S.S. Sastry. A Mathematical Introduction to Robotic Manipulation. CRC Press, 1994.

[8] T. D. Murphey and J. W. Burdick. A controllability test and motion planning primitives for overconstrained vehicles. In Proc. IEEE Int. Conf. on Robotics and Automation, Seoul, Korea, 2001.

[9] T. D. Murphey and J. W. Burdick. Issues in controllability and motion planning for overconstrained wheeled vehicles. In Proc. Int. Conf. Math. Theory of Networks and Systems (MTNS), Perpignan, France, 2000.

[10] A.D. Lewis, R.M. Murray, and J.W. Burdick. Nonholonomic mechanics and locomotion: the snakeboard example. In Proc. IEEE Int. Conf. on Robotics and Automation, pages 2391–2397, May 1994.